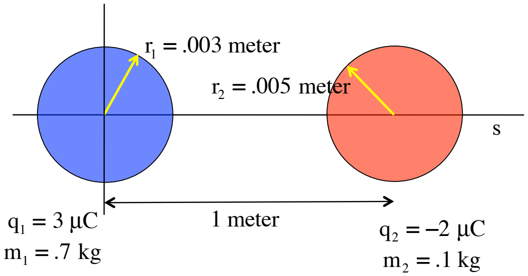
Problem 25.27

Two insulating spheres have their center-of-masses 1 meter apart. Released from rest, the two accelerate toward one another meeting at coordinate x. Determine x.



The first thing to notice is that because all of the forces acting in the system are internal to the system (the forces acting on the spheres are action/reaction couples), we can use *conservation of momentum* to begin generating relationships that are true. As such, we can write:

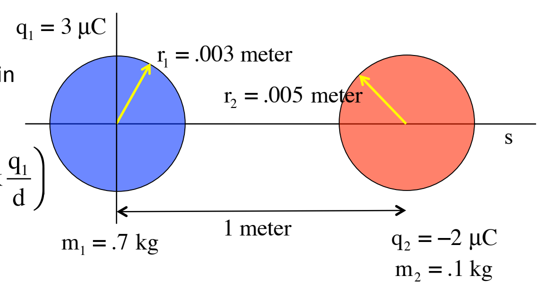
$$\sum p_{1,x} + \sum (F_{ext,x})(\Delta t) = \sum p_{2,x}$$

$$0 \quad + \quad 0 \quad = m_1 v_1 + m_2(-v_2)$$

$$\Rightarrow \quad v_2 = \frac{m_1}{m_2} v_1$$

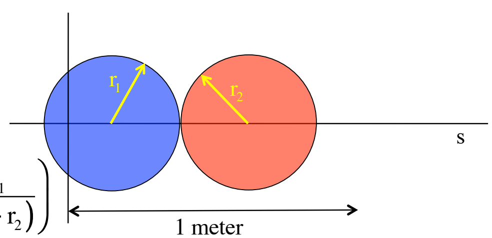
The initial potential energy U in the system is equal to:

$$U_1 = (-q_2) V_{\text{due to } q_1} = (-q_2)\left(k \frac{q_1}{d}\right)$$



The potential energy U in the system at collision time is equal to:

$$U_2 = (-q_2) V_{\text{due to } q_1} = (-q_2)\left(k \frac{q_1}{(r_1 + r_2)}\right)$$



So conservation of energy and substituting the expression for $v_2 = \frac{m_1}{m_2} v_1$ derived from the conservation of momentum, we get:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + k\frac{(-q_1)q_2}{d} + 0 = \left[\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2} v_1\right)^2\right] + k\frac{(-q_1)q_2}{(r_1 + r_2)}$$

$$\Rightarrow -k\frac{q_1 q_2}{d} + k\frac{q_1 q_2}{(r_1 + r_2)} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2} v_1\right)^2$$

$$\Rightarrow v_1 = \left[\frac{2 m_2 k q_1 q_2}{m_1(m_1 + m_2)}\left[\frac{1}{(r_1 + r_2)} - \frac{1}{d}\right]\right]^{1/2}$$

Putting in the numbers yields:

$$v_1 = \left[\frac{2 m_2 k q_1 q_2}{m_1(m_1 + m_2)}\left[\frac{1}{(r_1 + r_2)} - \frac{1}{d}\right]\right]^{1/2}$$

$$= \left[\frac{2(.7\text{kg})(9\text{x}10^9)(2\text{x}10^{-6}\text{C})(3\text{x}10^{-6}\text{C})}{(.1\text{kg})(.1 + .7)}\left[\frac{1}{8\text{x}10^{-3}} - \frac{1}{1}\right]\right]^{1/2}$$

$$= 10.8 \text{ m/s}$$

And:

$$v_2 = \frac{m_1}{m_2} v_1 = \frac{(.1\text{kg})(10.8\text{m/s})}{(.7\text{kg})} = 1.55 \text{ m/s}$$

b.) Would things change if the spheres had been conductors?

Yes. In that case, the electrons would have rearranged themselves so that the net charge on each ball would be located at the inner edge of their respective spheres. That would have changed the distance between the net charges to something less than $r_1 + r_2$ in the final position, elevating the velocity considerably.